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**Lab Manual # 10: QUBEStability and Second Order System Analysis**

***Objectives***

In this lab, we will learn about

* Stable, marginally stable, and unstable system
* Underdamped second-order system
* Calculation of Damping ratio and natural frequency
* Finding out the peak time and percent overshoot time-domain specifications

***Prerequisites***

* QUBE-Servo Integration laboratory experiment.
* Filtering laboratory experiment.

***Introduction***

**Servo Model:**

The QUBE-Servo voltage-to-speed transfer function is

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where is the model steady-state gain, is the model time constant, is the motor speed (i.e. speed of load disk), and is the applied motor voltage. If desired, you can conduct an experiment to find more precise model parameters, *K* and *τ,* for your particular servo (e.g., performing the Bump Test Modeling lab).

The voltage-to-position process transfer function can be found by integrating the above equation

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where is the load gear position.

**Stability:**

Definition for Bounded-Input Bounded-Output (BIBO) stability is:

1. A system is stable if every bounded input yields a bounded output.

2. A system is unstable if any bounded input yields an unbounded output. The stability of a system can be determined from its poles:

* Stable systems have poles only in the left-hand plane.
* Unstable systems have at least one pole in the right-hand plane and/or poles of multiplicity greater than 1 on the imaginary axis.
* Marginally stable systems have one pole on the imaginary axis and the other poles in the left-hand plane.

***Tasks:***

* Determine the stability of the voltage-to-speed servo system.
* Determine the stability of the voltage-to-position servo system.

**Calculations and comments on the stability**

Based on the VIs already designed in QUBE-Servo Integration and Filtering labs, design a VI that applies a step of 1V to the motor and reads the servo velocity and the position as shown in Figure 1. Configure the Simulation Loop to run for 2.5 seconds.

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(a) Front Panel



(b) Block Diagram

Figure 1: Measuring speed and position when applying a step

**Front Panel:**

* Based on the measured/observed *speed* response and the BIBO stability principle, comments on the stability of the system. How does this compare with your results from the pole analysis?

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* Based on the measured/observed *position* response and the BIBO stability principle, comments on the stability of the system. How does this compare with your results from the pole analysis?

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* Is there an input where the open-loop servo position response is BIBO stable? If so, modify the VI to include your input, test it on the servo, and show the position response. Based on this result, how could you define marginal stability? **Hint:** Try an impulse (short step) and compare the position for impulse response with the step response observed earlier.

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**Front Panel:**

***Conclusion***

**Second Order Step Response:**

The *standard second-order* transfer function has the form

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where is the natural frequency and *ζ* is the damping ratio. The properties of its response depend on the values of the parameters and *ζ* .

Consider a second-order system as shown in the above equation when subjected to a step input given by

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with a step amplitude of . The system response to this input is shown in Figure 2, where the red trace is the output response *y*(*t*) and the blue trace is the step input *r*(*t*).

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Figure 2 Standard second order step response

**Peak Time and Percent Overshoot:**

` The maximum value of the response is denoted by the variable and it occurs at a time . For a response similar to Figure 2, the percent overshoot is found using

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From the initial step time the time it takes for the response to reach its maximum value is

This is called the *peak time* of the system.

In a second-order system, the amount of overshoot depends solely on the damping ratio parameter, and it can be calculated using the equation

Diagram

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The peak time depends on both the damping ratio and natural frequency of the system, and it can be derived as

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Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

**Unity Feedback:**

The unity-feedback control loop shown in Figure 3 will be used to control the position of the QUBE-Servo. The input to the plant is the motor voltage . The reference is the desired motor position and the output is the actual position .

Diagram

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Figure 3 Unity Feedback Loop

The voltage-to-position transfer function is given by

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where is the load gear position.

The controller is denoted by *C* (*s*) whose value is unity.

The closed-loop transfer function of the QUBE-Servo position control from the reference input to the output using unity feedback as shown in Figure 3 is

***Tasks***

Design the VI as shown in Figure 4(b) implementing the unity feedback control. A step reference of 1 rad is applied at 1 second and the controller runs for 2.5 seconds. This is the desired position. To apply your step for a 2.5 seconds, set the *Final Time* of the Simulation Loop to 2.5 (instead of *inf* ). As shown in Figure 4, the unity feedback step response is “saved” using the Collector block from the *Control Design & Simulation* | *Simulation* | *Utilities* palette and displayed in an XY Graph. Use the cursors to take the measurements.

Graphical user interface, Excel

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(a) Front Panel

Diagram

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(b) Block Diagram

Figure 4: Unity feedback position control of QUBE-Servo

* Given the QUBE-Servo closed-loop equation under unity feedback system and the model parameters and which were already obtained from the bump modeling test, find the natural frequency and damping ratio of the system.
* Based on your obtained and *ζ* , also find the peak time and percent overshoot using the formulas.

Calculation:

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After comparison, we have

Diagram

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**Unity feedback QUBE-Servo step response (Front Panel)**

* Now evaluate the peak time and percent overshoot from the response and compare that with your theoretical results.

**Hint:** Use the cursor palette in the XY Graph to measure points off the plot.

***Conclusion***